THE EFFECT OF AN ELECTROMAGNETIC FIELD ON THE HEAT AND MASS TRANSFER WHICH OCCURS IN THE MELTING OF A ROTATING CYLINDER
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UDC 536.24.01;532.516;669.187

A qualitative analysis is given for the influence exerted by an electromagnetic field and the rotation of a cylinder on the velocity, pressure, and thickness of a film, on the temperature distribution within the film, and the yield of nonmetallic particles from the film.

A metallic cylinder with nonmetallic inclusions, in which a temperature $T_{c}$ is maintained, is placed within a liquid slag exhibiting a temperature $T_{S}$, with $T_{c}>T_{s}$, rotating at a constant angular velocity $\omega$.

Because of melting at the base of the cylinder, a liquid metallic film is formed and this film contains nonmetallic particles. The surface separating the solid cylinder and the liquid film is made to coincide with the origin of the axial coordinate $z=0$ and is replaced by a system of nonmetallic solid-particle sources with a concentration $C_{c}$. When we take into consideration the smallness of the film thickness $h(r)$ relative to the cylinder radius $R$ and the axisymmetricity of the problem, we find that the equations for the laminar boundary layer have the form [1]

$$
\begin{gather*}
V_{r} \frac{\partial V_{r}}{\partial r}+V_{z} \frac{\partial V_{r}}{\partial z}-\frac{V_{\varphi}^{2}}{r}=-\frac{1}{\rho} \frac{\partial P}{\partial r}+v \frac{\partial^{2} V_{r}}{\partial z^{2}}-\frac{\sigma \mu}{\rho} E_{z} H_{\varphi}-\frac{\sigma \mu^{2}}{\rho} V_{r} H_{甲}^{2}  \tag{1}\\
V_{r} \frac{\partial V_{\Phi}}{\partial r}+V_{z} \frac{\partial V_{\varphi}}{\partial z}+\frac{V_{r} V_{\varphi}}{r}=v \frac{\partial^{2} V_{\varphi}}{\partial z^{2}} \\
V_{r} \frac{\partial V_{z}}{\partial r}+V_{z} \frac{\partial V_{z}}{\partial z}=g-\frac{1}{\rho} \frac{\partial P}{\partial z}+v \frac{\partial^{2} V_{z}}{\partial z^{2}}-\frac{\sigma \mu^{2}}{\rho} V_{z} H_{\varphi}^{2} \\
\frac{\partial V_{r}}{\partial r}+\frac{\partial V_{z}}{\partial z}+\frac{V_{r}}{r}=0  \tag{2}\\
V_{r} \frac{\partial T}{\partial r}+V_{z} \frac{\partial T}{\partial z}=a \frac{\partial^{2} T}{\partial z^{2}}+\frac{\rho}{\rho C_{p}}\left[E_{z}^{2}+\mu^{2}\left(V_{r}^{2}+V_{z}^{2}\right) H_{\varphi}^{2}\right]  \tag{3}\\
V_{r} \frac{\partial C}{\partial r}+V_{z} \frac{\partial C}{\partial z}=D \frac{\partial^{2} C}{\partial z^{2}}  \tag{4}\\
 \tag{5}\\
\\
\end{gather*}
$$

The boundary conditions are assumed to be the following:

$$
\begin{gather*}
z=0, \quad V_{r}=0, \quad V_{\Phi}=r \omega, \quad V_{z}=V_{\mathrm{m}}, \quad T=T_{\mathrm{c}}, \quad C=C_{\mathrm{c}} ;  \tag{6}\\
z=h(r), \quad V_{r} \frac{d h}{d r}=V_{z}, \quad T==T_{\mathrm{s}} \tag{7}
\end{gather*}
$$

where $V_{m}$ is the rate of cylinder melting.

All-Union Scientific-Research Petroleum Geology Exploration Institute, Arkhangel'sk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 56, No. 2, pp. 307-312, February, 1989. Original article submitted August 24, 1987.

The electric-charge density $\rho^{\prime}$ within the $f i l m$ is assumed to be small, $\rho^{\prime} \sim 0$, while the conductivity $\sigma$, conversely, is assumed to be rather large, $\sigma \sim O\left(1 / h^{2}\right)$.

Between the current density $j$ and the field strength $E_{z}$ we assume the simplest of relationships, i.e.,

$$
\begin{equation*}
j=\sigma E_{z}, \tag{8}
\end{equation*}
$$

where we assume $j=$ const in the film. Then Eq. (5) determines the strength of the magnetic field within the film

$$
\begin{equation*}
H_{\varphi}=j r . \tag{9}
\end{equation*}
$$

The velocity components $V_{T}, V_{\Phi}$, and $V_{Z}$, and the pressure $P$ are sought in the form

$$
\begin{align*}
V_{r}=\sum_{n=0}^{\infty} f_{n}(\eta) \xi^{n}, \quad V_{\varphi} & =\sum_{n=0}^{\infty} \varphi_{n}(\eta) \xi^{n}, \quad V_{z}=\sum_{n=0}^{\infty} \psi_{n}(\eta) \xi^{n}  \tag{10}\\
P & =\sum_{n=0}^{\infty} x_{n}(\eta) \xi^{n} .
\end{align*}
$$

Here and below we have adopted the following notation: $\xi=z / R, \eta=r / R, V_{e}=\sigma \mu^{2} j^{2} R^{3} / \rho$ has the dimensionality of velocity; $R e_{\omega}=\omega R^{2} / \nu, R e_{z}=R V_{m} / v$ are the Reynolds numbers for the rotational and translational motions of the film, respectively; $\mathrm{Fr}=\mathrm{g} / \mathrm{R} \omega^{2}$ is the Froude number.

Substituting relationships (10) into the Navier-Stokes equations (1) and inequality (2), and taking into account (8)-(9), for the functions $f_{n}(\eta), \varphi_{n}(\eta), \psi_{n}(\eta), X_{n}(\eta)$, we obtain an infinite system of equations. After integration of the equations of the system with consideration of boundary conditions (6), we note that the functions $f_{n}, \varphi_{n}, \psi_{n}, X_{n}$ assume the form of nonorthogonal polynomials with respect to $\eta$.

In conclusion, we derive the following laws to govern the distribution of velocities and pressures within the film, retaining only the senior terms of series (10):

$$
\begin{align*}
& V_{r}=\frac{1}{3} R \omega \operatorname{Re}_{\omega}^{2} \eta^{2} \xi^{3}\left(1+\frac{1}{2} \operatorname{Re}_{z} \xi-\frac{1}{4} \operatorname{Re}_{\omega} \eta \xi+\ldots\right)- \\
& \frac{1}{4} V_{\mathrm{e}} \operatorname{Re}_{z} \eta^{3 \xi}\left(1+\frac{1}{2} \operatorname{Re}_{z} \xi+\frac{1}{6} \operatorname{Re}_{z}^{3} \xi^{2}+\frac{1}{24} \operatorname{Re}_{z}^{3} \xi^{3}+\ldots\right)- \\
& -\frac{1}{8} \frac{V_{\mathrm{e}}^{2}}{V_{\mathrm{m}}^{2}} \operatorname{Re}_{z}^{2} \eta^{5} \xi^{3}\left(\frac{1}{3}+\frac{1}{8} \operatorname{Re}_{z} \xi+\ldots\right) \text {, }  \tag{11}\\
& V_{z}=V_{\mathrm{m}}+\frac{1}{2} V_{\mathrm{e}} \mathrm{Re}_{z} \eta^{2 \xi^{2}}\left(1+\frac{1}{3} \mathrm{Re}_{z} \xi+\frac{1}{12} \mathrm{Re}_{z}^{2} \xi^{2}+\right. \\
& \left.\frac{1}{60} \operatorname{Re}_{z}^{3} \xi^{3}+\ldots\right)+\frac{1}{16} \frac{V_{\mathrm{e}}^{2}}{V_{\mathrm{m}}} \operatorname{Re}_{z}^{2} \eta^{4} \xi^{4}\left(1+\frac{3}{10} \operatorname{Re}_{z} \xi+\ldots\right)- \\
& -\frac{\omega R}{4} \operatorname{Re}_{\omega}^{2} \eta \xi^{\xi}\left(1+\frac{2}{5} \operatorname{Re}_{z} \xi-\frac{4}{15} \operatorname{Re}_{\omega} \eta \xi+\ldots\right),  \tag{12}\\
& V_{\varphi}=\omega R \eta\left[1-\operatorname{Re}_{\omega} \eta \xi\left(1+\frac{1}{2} \operatorname{Re}_{z} \xi+\frac{1}{6} \operatorname{Re}_{z}^{2} \xi^{2}+\right.\right. \\
& \left.\left.\frac{1}{24} \operatorname{Re}_{z}^{3} \xi^{3}+\ldots\right)-\frac{1}{12} \frac{V \mathrm{e}}{V_{\mathrm{m}}} \operatorname{Re}_{z}^{2} \eta^{2 \varepsilon} \xi^{3}\left(1+\frac{1}{2} \operatorname{Re}_{z} \xi+\frac{1}{2} \operatorname{Re}_{\omega} \eta \xi+\ldots\right)\right] \text {, }  \tag{13}\\
& P=P_{0}+\rho(\omega R)^{2}\left[\frac{1}{2}\left(1-\frac{\mu j^{2}}{\rho \omega^{2}}\right) \eta^{2}+\xi \mathrm{Fr}-\frac{2}{3} \mathrm{Re}_{\omega} \eta \xi^{3}+\frac{1}{4}\left(\frac{V_{\mathrm{e}}}{\omega R}\right)^{2} \mathrm{Re}_{z} \eta^{4 \xi 3}+\ldots\right] . \tag{14}
\end{align*}
$$

Since the ratio of the actual film-thickness values $h$ and the cylinder radius $R$ are on the order of $10^{-3}$ and the numbers $R e_{\omega}$ and particularly $R e_{z}$ are small because of the large viscosity $v\left(\operatorname{Re}_{z} \sim 1, \operatorname{Re}_{\omega}<200\right)$, convergence of series (11)-(14) is obvious.

The equation $V_{r}(d h / d r)=V_{Z}$ in conditions (7) expresses the equality of the normal li-quid-film and surrounding liquid-slag velocity components and, consequently, can serve to determine the shape of the film surface. After integration of relationship $V_{r}(r, h)(d h / d r)=$ $V_{Z}(r, h)$ we obtain the following equation for the determination of film thickness:

$$
\begin{equation*}
\sum_{n=1}^{\infty} \frac{f_{n}(r)}{n+1} h^{n+1}=\frac{r}{2} V_{\mathrm{m}} \tag{15}
\end{equation*}
$$

or on retention only of the senior terms in the functions $f_{n}(r)$ we will have

$$
\begin{equation*}
h(\eta)=R\left[\frac{1}{8 \alpha-\eta^{3}}\left(\frac{6}{\operatorname{Re}_{\mathrm{e}}} \eta+\left(\frac{12}{\operatorname{Re}_{\mathrm{e}}}\left(\frac{32 \alpha}{\eta}-\eta^{2}\right)\right)^{1 / 2}\right)\right]^{1 / 2} \tag{16}
\end{equation*}
$$

where $R e_{e}=R V_{e} / v$ is the analog of the Reynolds number; $\alpha=\operatorname{Re}_{\omega}{ }^{3} /\left(\operatorname{Re}_{2} \operatorname{Re}_{e}{ }^{2}\right) ; 0<\eta \leq 1$.
Of interest is the surface at which $V_{Z}=0$. In approximate terms, the equation for such streamlines will have the form

$$
\begin{equation*}
h_{\mathrm{c}}(\eta)=R\left[\frac{1}{4 \alpha-\eta^{3}}\left(\frac{4}{\mathrm{Re}_{\mathrm{e}}} \eta+\left(\frac{64 \alpha}{\eta \mathrm{Re}_{\mathrm{e}}}\right)^{1 / 2}\right)\right]^{1 / 2} \tag{17}
\end{equation*}
$$

Bearing in mind that $\alpha>1$, we obtain the ratio $h_{c}(\eta) / h(\eta)$ which is on the order of 0.9 . With

$$
\begin{equation*}
\eta_{b}=\left[\left(-\frac{3 V \mathrm{~m}^{2}}{\beta^{2} \mathrm{Re}_{z}}+\sqrt{\frac{9 V_{\mathrm{m}}^{2}}{\beta^{\mu} \mathrm{Re}_{z}^{2}}+\frac{8 R \omega \mathrm{Re}_{\omega}^{2} V_{\mathrm{m}}}{\mathrm{Re}_{z}^{2}}}\right) \frac{1}{V_{\mathrm{e}}}\right]^{2 / 3} \tag{17a}
\end{equation*}
$$

reverse flows in the radial direction appear at the film surface, which is explained by the predominant influence of electromagnetic forces over inertial forces.

With small current-density values $\eta_{b} \gg 1$. The larger the value of $j$, the closer to the axis of rotation do the reverse flows begin.

In order to solve the heat and mass transfer equations (3)-(4), we will limit ourselves to the following form of the velocity components:

$$
\begin{equation*}
V_{r}=\frac{1}{3} R \omega \operatorname{Re}_{\omega}^{2} \eta^{2} \xi^{3}-\frac{1}{4} V_{\mathrm{e}} \mathrm{Re}_{z} \eta^{3 \xi}, \quad V_{z}=V_{\mathrm{m}}-\frac{1}{4} R \omega \mathrm{Re}_{\omega}^{2} \eta \xi^{4}+\frac{1}{2} V_{\mathrm{e}} \mathrm{Re}_{z} \eta^{2 \xi \xi^{2}} \tag{18}
\end{equation*}
$$

Correspondingly, for the film thickness

$$
\begin{equation*}
h(\eta)=\beta R \eta^{-1 / 4} \tag{19}
\end{equation*}
$$

where

$$
\beta=\left(\frac{6}{\alpha \operatorname{Re}_{\mathrm{e}}}\right)^{1 / 4}
$$

Then relationships (3)-(4) assume the form

$$
\begin{gathered}
{\left[\frac{1}{3} \operatorname{Re}_{\omega}^{2} \eta^{2} \xi^{9}-\frac{1}{4} \frac{\operatorname{Re} \mathrm{Re}_{z}}{\operatorname{Re}_{\omega}} \eta^{3 \xi}\right] \frac{\partial T}{\partial \eta}+\left[\frac{\mathrm{Re}_{z}}{\operatorname{Re}_{\omega}}-\frac{1}{4} \mathrm{Re}_{\omega}^{2} \eta \xi^{4}+\right.} \\
\left.\frac{1}{2} \frac{\operatorname{Re}_{\mathrm{e}} \mathrm{Re}_{z}}{\operatorname{Re}_{\omega}} \eta^{2 \xi^{2}}\right] \frac{\partial T}{\partial \xi}=\frac{a}{v \mathrm{Re}_{\omega}} \frac{\partial^{2} T}{\partial \xi^{2}}+\frac{j^{2}}{\rho \omega \sigma c_{p}}+
\end{gathered}
$$

$$
\begin{align*}
& \left.\left.\frac{1}{2} \frac{R E e^{R e_{z}}}{R e_{\omega}} \eta^{\eta * \xi_{2}}\right)^{2}\right] \text {, }  \tag{20}\\
& \left(\frac{1}{3} \operatorname{Re}_{\omega}^{2} \eta^{2 \xi} \xi^{3}-\frac{1}{4} \frac{\operatorname{Rte}^{R} \mathrm{Re}_{z}}{\operatorname{Re}_{\omega}} \eta^{3 \xi}\right) \frac{\partial C}{\partial \eta}+\left(\frac{\mathrm{Re}_{z}}{\operatorname{Re}_{\omega}}-\frac{1}{4} \operatorname{Re}_{\omega}^{2} \eta \eta_{\xi}^{\xi}+\right. \\
& \left.+\frac{1}{2} \underset{\operatorname{Re}_{e} \operatorname{Re}_{z}}{\operatorname{Re}_{\omega}} \eta^{2} \xi^{2}\right) \frac{\partial C}{\partial \xi}=\frac{D}{v \operatorname{Re}_{a j}} \frac{\partial^{2} C}{\partial \xi^{2}} \tag{21}
\end{align*}
$$

with boundary conditions

$$
\begin{equation*}
\xi=0, \quad T=T_{\mathrm{c}} . \quad C=C_{\mathrm{c}} . \quad \xi-\beta \eta^{-1 / 4}, \quad T=T_{\mathrm{s}} \tag{22}
\end{equation*}
$$

and any Peclet and Prandtl numbers. The solution of Eqs. (20)-(21) under conditions (22) will be sought in the following form [2]:

$$
\begin{gather*}
T=T_{\mathrm{c}}-\left(T_{\mathrm{c}}-T_{\mathrm{s}}\right)\left[\frac{\xi}{\beta} \eta^{1 / 4}+\left(\xi-\frac{\xi^{2}}{\beta} \eta^{1 / 4}\right) F(\eta)\right],  \tag{23}\\
C=C_{\mathrm{c}}\left[1+\left(\xi+\xi^{2}\right) \Phi(\eta)\right] . \tag{24}
\end{gather*}
$$

To find the functions $F(\eta)$ and $\Phi(\eta)$ by the method of reduction to ordinary differential equations, we will derive the following relationships:

$$
\begin{align*}
& {\left[\frac{1}{84} \gamma \beta \eta-\frac{1}{240} \gamma^{3} \eta^{2}, 5\right] \frac{d F}{d \eta}+\left[\frac{1}{112} \gamma \beta-\frac{1}{160} \gamma^{3} \eta^{1,5}+\right.} \\
& \left.\frac{a}{3 v \mathrm{Re}_{\mathrm{w}}} \eta^{1 / 4}\right] F=\frac{j^{2} \beta}{6 p \omega \sigma c_{p}\left(T_{\mathrm{s}}-T_{\mathrm{c}}\right)}-\frac{1}{6} \frac{\gamma}{\mathrm{Re}_{\mathrm{e}}} \eta^{1 / 4}+ \\
& \frac{1}{42} \gamma \eta^{1 / 4}-\frac{7}{320} \gamma^{2} \eta^{7 / 4}+\frac{V e}{c_{p}\left(T_{s}-T_{\mathrm{c}}\right)}\left[\frac{1}{18} \frac{\gamma^{2}}{\beta} \eta^{4.5}-\right. \\
& \frac{1}{42} \gamma^{2} \beta \eta^{6}+\frac{1}{320} \gamma^{2} \beta^{2} \eta^{7,5}+\frac{1}{6 \mathrm{R}_{\mathrm{e}} \mathrm{e}} \gamma^{2} \beta \eta^{2}+\frac{9}{440} \gamma^{2} \beta \eta^{2}+  \tag{25}\\
& \left.\frac{1}{168} \gamma^{2} \beta^{5} \eta^{5}-\frac{1}{14 \mathrm{Re}_{\mathrm{e}}} \gamma^{2} \beta \eta^{2}+\frac{1}{20 \mathrm{Re}_{\mathrm{e}}} \gamma^{2} \beta^{3} \eta^{3,5}-\frac{1}{48} \gamma^{2} \beta^{3} \eta^{3,5}\right], \\
& {\left[\frac{1}{3} \mathrm{Re}_{\omega}^{2} \beta^{4}\left(\frac{1}{6} \eta^{1 / 2}+\frac{2}{7} \beta \eta^{1 / 4}+\frac{1}{8} \beta^{2}\right) \eta-\frac{1}{4} \gamma \beta^{2}\left(\frac{1}{4} \eta+\right.\right.} \\
& \left.\left.\frac{2}{5} \beta \eta^{3 / 4}+\frac{1}{6} \beta^{2} \eta^{1 / 2}\right) \eta^{2}\right] \frac{d \Phi}{d \eta}+\left[\frac { \operatorname { R e } _ { z } } { \operatorname { R e } _ { \omega } } \left(\frac{1}{2} \eta^{1 / 2}+\beta \eta^{1 / 4}+\right.\right. \\
& \left.\frac{1}{2} \beta^{2}\right)-\frac{1}{4} \operatorname{Re}_{\omega}^{2} \beta^{4}\left(\frac{1}{6} \eta^{1 / 2}+\frac{3}{7} \beta \eta^{1 / 4}+\frac{1}{4} \beta^{2}\right)+ \\
& \frac{1}{2} \gamma \beta^{2}\left(\frac{1}{4} \eta+\frac{3}{5} \beta \eta^{3 / 4}+\frac{1}{3} \beta^{2} \eta^{1 / 2}\right) \eta- \\
& \left.\frac{2 D}{\nu \operatorname{Re}_{\mathrm{w}}}\left(\frac{1}{2} \eta^{1 / 2}+\frac{1}{3} \beta \eta^{1 / 4}\right)\right] \Phi=0,
\end{align*}
$$

where $\gamma=\operatorname{Re}_{e} \mathrm{Re}_{Z} / \mathrm{Re}_{\omega}$.
Equation (25) is satisfied by a series of the form

$$
\begin{equation*}
F(\eta)=\sum_{n=0}^{\infty} B_{n}\left(\eta^{1 / 4}\right)^{n}, \tag{27}
\end{equation*}
$$

where

$$
\begin{align*}
& B_{0}=\frac{56 j^{2} \mathrm{Re}_{\omega}}{3 \rho \omega \sigma c_{p}\left(T_{\mathrm{s}}-T_{\mathrm{c}}\right) \mathrm{Re}_{\mathrm{e}} \mathrm{Re}_{z}} ; \\
& B_{1}=\frac{336}{4}\left[-\frac{a B_{0}}{3 v \gamma \beta \operatorname{Re}_{\omega}}-\frac{1}{42 \beta}+\frac{1}{6 \beta \operatorname{Re}_{\mathrm{e}}}\right] ; \quad B_{k}=\frac{336}{k+3}\left[-\frac{a B_{h-1}}{3 v \gamma \beta \mathrm{Re}_{\omega}}\right] \text {, } \\
& k=2,3,4,5 ; \\
& B_{e}=\frac{336}{9}\left[-\frac{a B_{5}}{3 v \gamma \operatorname{Re}_{\omega}}+\frac{B_{0} \beta^{2}}{160}\right] ; \quad B_{7}=\frac{336}{10}\left[-\frac{a B_{6}}{3 v \gamma \beta \mathrm{Re}_{\omega}}+\right. \\
& \left.\frac{7 B_{1} \beta^{2}}{960}-\frac{7 \beta}{320}\right] ; \quad B_{8}=\frac{336}{11}\left[-\frac{a B_{7}}{3 v \gamma \beta \mathrm{Re}_{6}}+\frac{8 B_{2} \beta^{2}}{960}+\right. \\
& \left.\frac{V_{\mathrm{e}} \mathrm{Y}}{c_{p}\left(T_{\mathrm{s}}-T_{\mathrm{c}}\right)}\left(\frac{1}{6 \mathrm{Re}_{\mathrm{e}}^{\mathrm{e}}} v+-\frac{9}{440}+\frac{1}{14 \mathrm{Rf}_{\mathrm{e}}}\right)\right] ;  \tag{28}\\
& B_{14}=\frac{336}{17}\left[-\frac{a B_{1 \mathrm{~s}}}{3 v \gamma \beta \mathrm{Re}_{\omega}}+\frac{14 B_{8} \beta^{2}}{960}+\frac{V_{e} \nu \beta^{2}}{c_{p}\left(T_{\mathrm{s}}-T_{\mathrm{c}}\right)}\left(\frac{1}{20 \operatorname{Re}_{\mathrm{e}}}-\frac{1}{48}\right)\right] ; \\
& B_{18}=\frac{336}{21}\left[-\frac{a B_{17}}{3 v \gamma \beta e_{\omega}}+\frac{18 B_{12} \beta^{2}}{960}+\frac{V_{\mathrm{e}} \gamma}{18 \beta^{2} c_{p}\left(T_{\mathrm{s}}-T_{\mathrm{c}}\right)}\right] \text {, } \\
& B_{20}=\frac{336}{23}\left[-\frac{a B_{19}}{3 v \gamma \beta \mathrm{e}_{\mathrm{\omega}}}+\frac{20 B_{14} \beta^{2}}{960}+\frac{V_{\mathrm{e} \ell} \gamma \beta^{4}}{168 c_{p}\left(T_{\mathrm{s}}-T_{\mathrm{c}}\right)}\right] \text {, } \\
& B_{24}=\frac{336}{27}\left[-\frac{a B_{23}}{3 v \gamma \beta \mathrm{Re}_{6}}+\frac{24 B_{18} \beta^{2}}{960}--\frac{V_{\mathrm{e}} \psi}{42 c_{p}\left(T_{\mathrm{s}}-T_{\mathrm{c}}\right)}\right] \text {, } \\
& B_{30}=\frac{336}{33}\left[-\frac{a B_{2 g}}{3 v \gamma \beta \mathrm{e}_{\omega}}+\frac{30 B_{24} \beta^{2}}{960}+\frac{V_{\mathrm{e} 7 \beta}}{320 c_{p}\left(T_{\mathrm{s}}-T_{\mathrm{c}}\right)}\right] \text {, } \\
& B_{k}=\frac{336}{k+3}\left[-\frac{a B_{k-1}}{3 v \gamma \beta \mathrm{Re}_{\omega}}+\frac{k B_{k-6} \beta^{2}}{960}\right], \quad k \geqslant 9, \quad k \neq 14,18,20,24,30 .
\end{align*}
$$

The function $\Phi(\eta)$ in Eq. (26) is determined by the following approximate relationship:

$$
\begin{gather*}
\Phi(\eta)=A \eta^{\frac{3}{8}-\frac{1}{2 R e_{e}}}\left[\left(\eta^{1 / 2}+\frac{12}{7} \beta \eta^{1 / 4}+\frac{3}{4} \beta^{2}\right)^{\left.-\frac{3}{8}+\frac{1}{4 \mathrm{Re}_{\mathrm{e}}}+\frac{3 D}{2 \nu \mathrm{Rf} \mathrm{e}_{\mathrm{e}}^{j}}\right] \times}\right.  \tag{29}\\
\exp \left(-\frac{4 D \beta}{7 v \mathrm{Re}_{\mathrm{e}}^{3}} \operatorname{arctg} \frac{14}{\sqrt{3}}\left(\frac{\eta^{1 / 4}}{\beta}+\frac{6}{7}\right)\right),
\end{gather*}
$$

where

$$
\beta=\left(\frac{6 R \epsilon_{e} R e_{z}}{\operatorname{Re}_{\omega}^{3}}\right)^{1 / 4}
$$

The integration constant A can either be determined experimentally, or from some additional condition at the film boundary. The temperature gradient $\partial T / \partial \xi$ at the boundary of separation between the solid cylinder and the liquid film is equal to

$$
\begin{equation*}
\frac{\partial T}{\partial \xi}(\eta, 0)=\left(T_{\mathrm{s}}-T_{\mathrm{c}}\right)\left(\frac{1}{\beta} \eta^{1 / 4}+F(\eta)\right) \tag{30}
\end{equation*}
$$

and characterizes the qualitative form of the end surface of the cylinder. The particle concentration gradient $\partial \mathrm{C} / \partial \xi$ at the boundary of separation between the liquid film and the liquid slag has the form

$$
\begin{equation*}
\frac{\partial C}{\partial \xi}(\eta, h(\eta))=C_{c} \phi(\eta)\left(1+2 \beta \eta^{-1 / 4}\right) . \tag{31}
\end{equation*}
$$

Analysis of relationship (29) shows that $\mathrm{Re}_{\mathrm{e}}$ must be no less than $4 / 3$, in order to prevent $\Phi(\eta)$ from becoming infinitely large. In order to prevent $\partial C / \partial \xi$ in relationship (31) from becoming infinitely large, we must have $R e_{e}$ with a value of no less than 4. Consequently, when $4 / 3 \leq \operatorname{Re}_{e}<4$ the principal mass of nonmetallic particles departs from the film and enters the slag near the axis of cylinder rotation. When $R e_{e}=4$, we will observe a rela-
tively uniform yield of particles over the entire surface of the film. When $R e_{e}>4$, the yield of particles shifts in the direction of the edge of the cylinder base. With very large values for $\mathrm{Re}_{\mathrm{e}}$ the yield of particles will be noted primarily at the edge of the base, provided that $\mathrm{Re}_{\omega}$ is not large. If $\mathrm{Re}_{e}$ and $\mathrm{Re}_{\omega}$ are of the same order of magnitude, with both quantities rather large, the particle yield will follow the 3/16 law, i.e.,

$$
\begin{equation*}
\frac{\partial C}{\partial \xi}(\eta, h(\eta))=A C_{\mathbf{c}} \eta^{3 / 16} . \tag{32}
\end{equation*}
$$

## NOTATION

$\mathrm{r}, \mathrm{q}, \mathrm{Z}, \mathrm{cylindrical}$ coordinates; $\mathrm{V}_{\mathrm{r}}, \mathrm{V}_{\mathrm{p}}, \mathrm{V}_{\mathrm{Z}}$, velocity components; P , pressure; $\mathrm{E}_{\mathrm{Z}}$, $H_{\varphi}$, components of the electric and magnetic field strengths; $\omega$, angular velocity; $T$, temperature; $h$, film thickness; $C$, concentration; $\rho$, density of the liquid; $v$, kinematic viscosity; $g$, acceleration of the force of gravity; $\sigma$, conductivity; $j$, current density; $\mu$, magnetic permeability; $c_{p}$, specific heat capacity; $a$, coefficient of thermal diffusivity; D, diffusion coefficient.

## LITERATURE CITED

1. K. I. Strakhovich, Hydro- and Gasdynamics [in Russian], Moscow (1980).
2. L. V. Kontorovich and V. I. Krylov, Approximate Methods of Higher Analysis [in Russian], Moscow-Leningrad (1952).

TWO-MODEL ITERATION METHOD FOR THE SOLUTION OF
an Inverse boundary heat-EXchange problem
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A method is proposed for restoration of the boundary condition, with an iterative correction of the initial data used in this method, involving the utilization of both exact and approximate heat-transfer models.

In constructing installations the design and regime parameters are greatly affected by the projection of heat conditions, this latter significantly dependent on the reliability of processing experimental information as well as on the quality of the mathematical models employed. One of the latest trends in the theory and practice of heat research is based on the application of the principles of inverse heat-exchange problems. The development of effective regularization methods [1, 2] has made it possible to overcome the errors in the formulations of the inverse heat-exchange problems, as had already been noted in [3]. The principle of iteration regularization in extremum formulation of the solution for the inverse heat-exchange problem as proposed by Alifanov and introduction into our examination of the Tikhonov stabilizing functional makes it possible to avoid fluctuations in the restoration parameters in the presence of random errors in the initial temperatures and in errors of approximating differential equations with difference equations.

Formulations of inverse problems, just as in the formulation of direct problems, presuppose the representation of the real heat-exchange process in some mathematical form to express significant factors and interrelationships in the phenomenon being studied. Contemporary methodology of modeling rejects the concept of a "single process - single model." As is validly noted in [4], an entire family of models may be used for any given phenomenon, with these models differing, in particular, in the number of factors which they take into

Kuibyshev Polytechnic Institute, Kuibyshev. Translated from Inzhenerno-Fizicheskii Zhurnal, Vo1. 56, No. 2, pp. 313-319, February, 1989. Original article submitted August 6, 1987.

